CMPSCI 691AD – General Purpose Computation on the GPU

Spring 2009

Lecture 16: Space Partitioning

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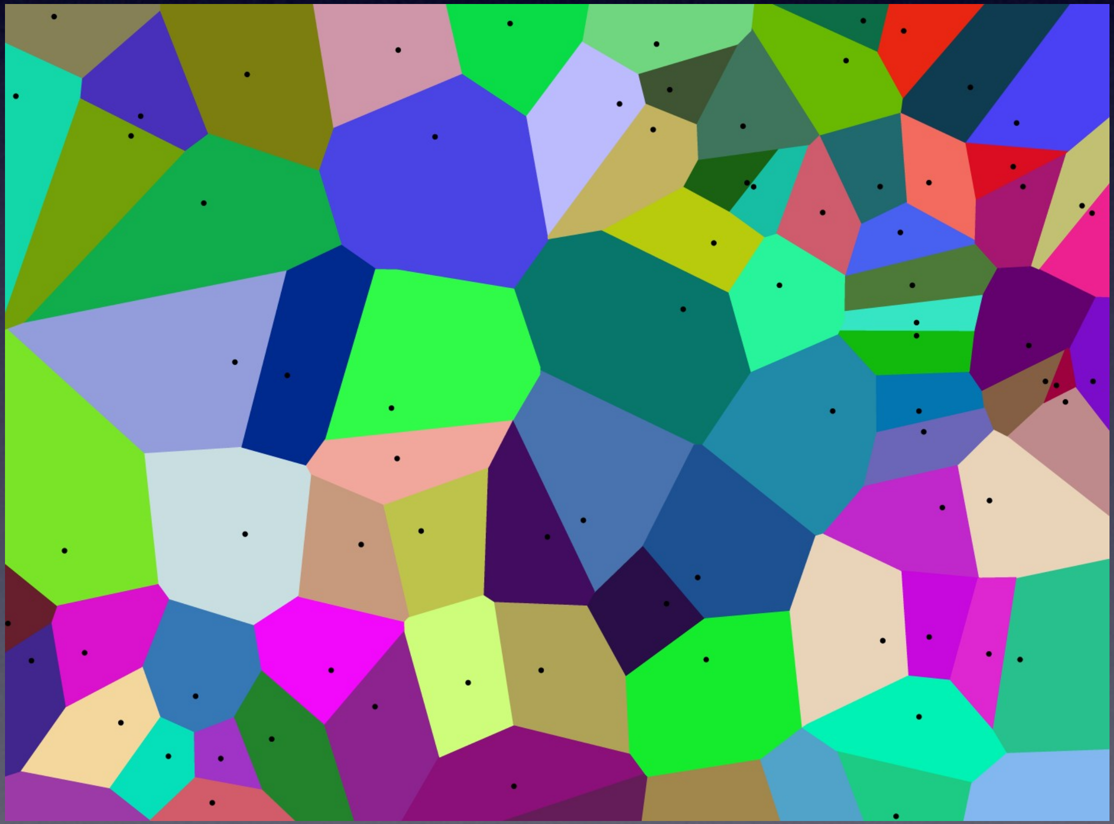
Topics

- Voronoi Diagram
- Uniform Grid
- KD-Tree
- kNN Search

Voronoi Diagram

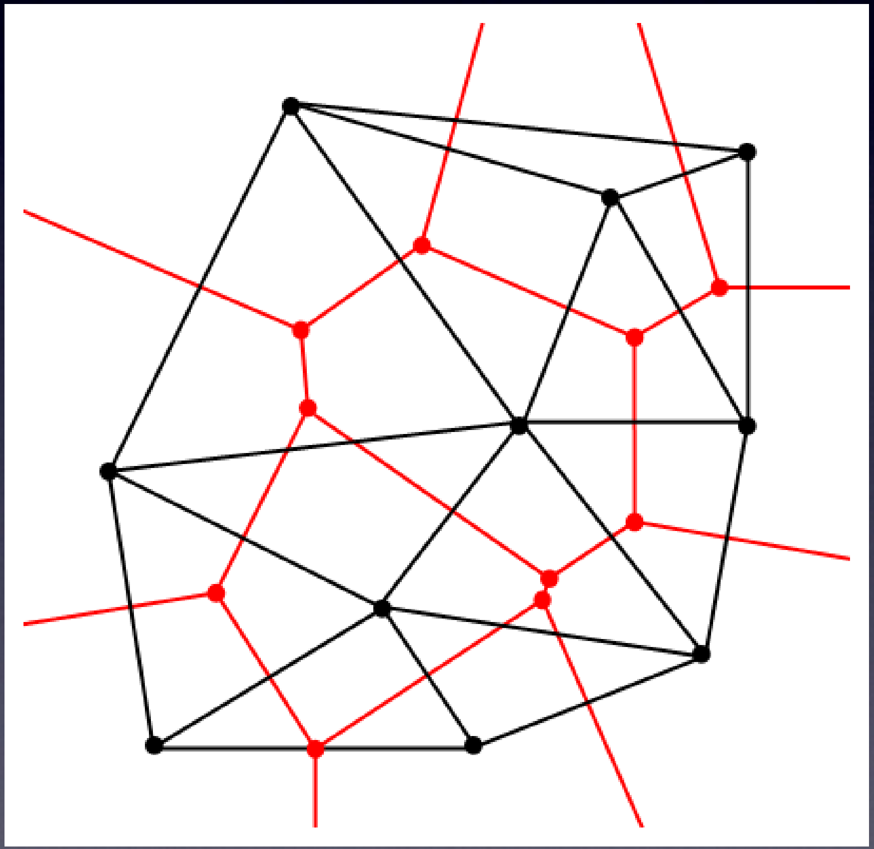
- Given a set of points S in Euclidean space, find a partitioning of space into cells, such that any point within a cell is closer to one point in S than any other point in S.
- Example:



Voronoi Diagram

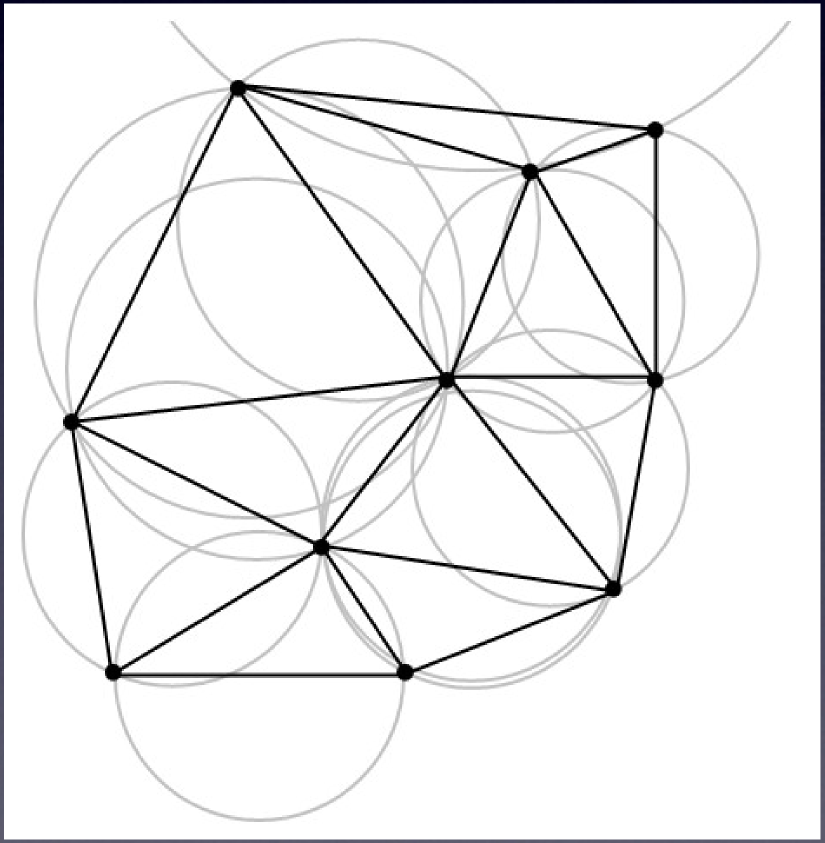
- The dual graph of a Voronoi diagram is a Delaunay triangulation

  - Black dots: point set S
  - Red lines: Voronoi diagram
  - Black lines: Delaunay triangles



Delaunay Triangulation

- A triangulation of point set S in Euclidean space, such as no point in S is inside the circumcircle of any triangle.



Voronoi Diagram

- Computing Voronoi Diagram:
  - Bowyer-Watson algorithm
  - Fortune's algorithm
  - Lloyd's algorithm

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- Let's think about a brute-force numerical solution:

  - Focus on 2D, and compute the solution on an NxN image (solution space)
  - For every pixel in the image, classify it to its closest Voronoi center.

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  - Lloyd's algorithm
- Let's think about a brute-force numerical solution:

  - Focus on 2D, and compute the solution on an NxN image (solution space)
  - For every pixel in the image, classify it to its closest Voronoi center.
- But this sounds very expensive, especially if the point set is large!

Space Partitioning

- Question: given a point set S, and an arbitrary point p in space, how to quickly find the closest point in S to p?
- This is called the nearest neighbor (NN) search problem.

Space Partitioning

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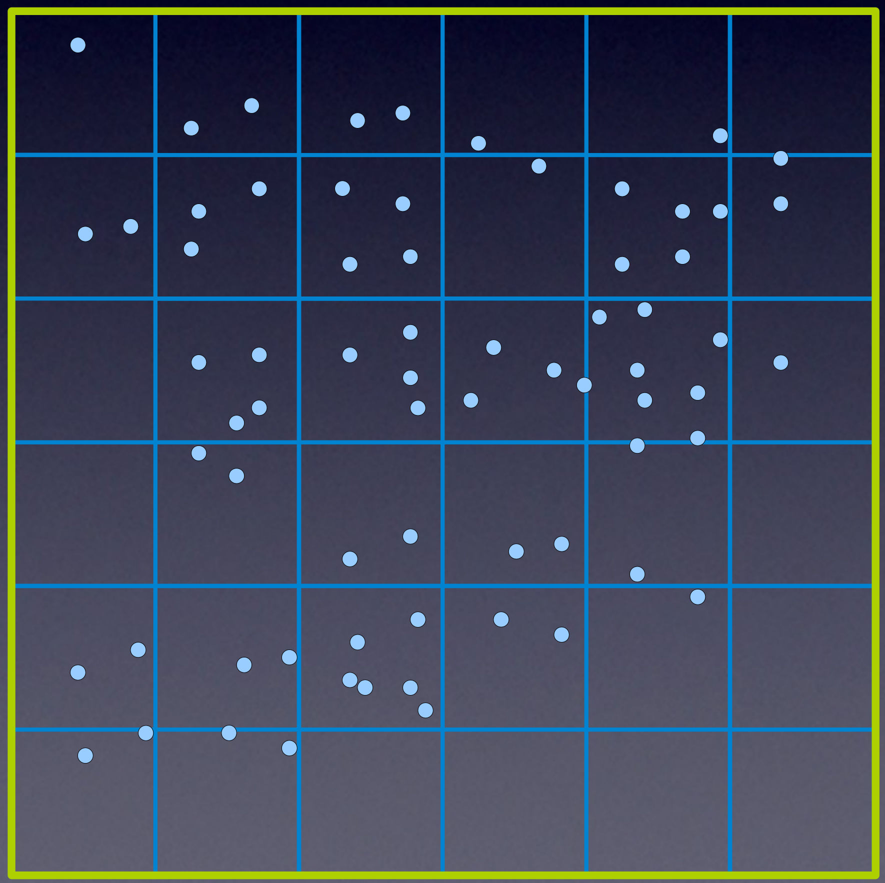
- This is called the nearest neighbor (NN) search problem.
- A key to make it efficient is to quickly eliminate groups of points that can not possibly be the solution.
  - Use space partitioning data structures!

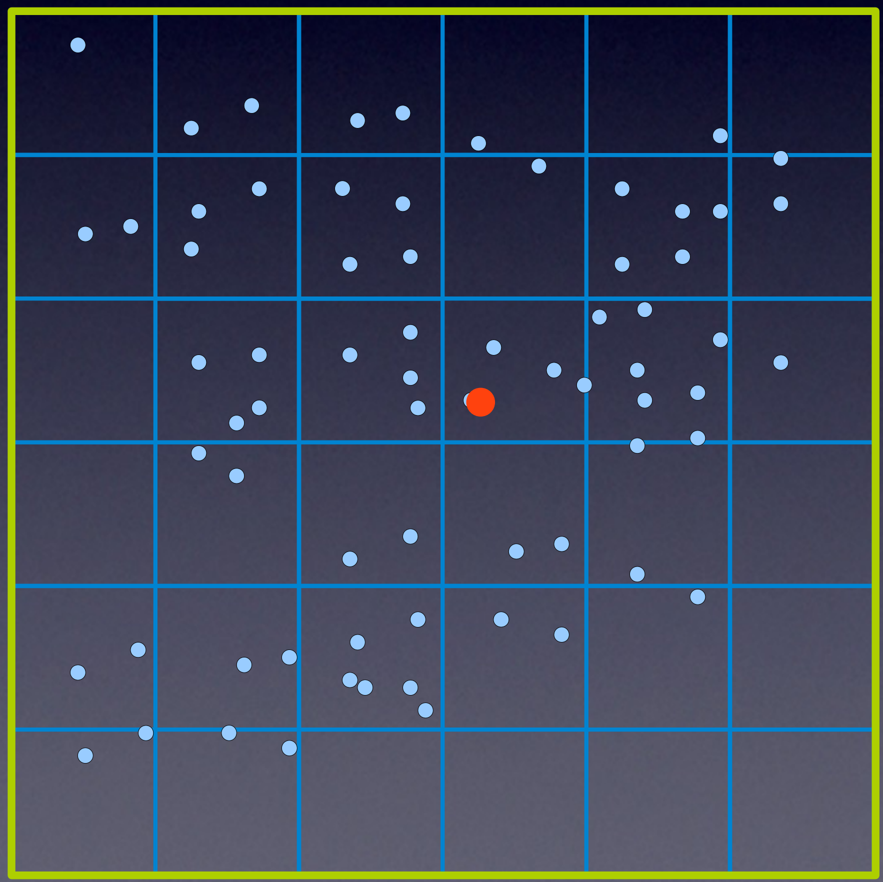
Uniform Grid

- Uniformly partition the space into cells
  - Keep track of the list of points inside each cell.



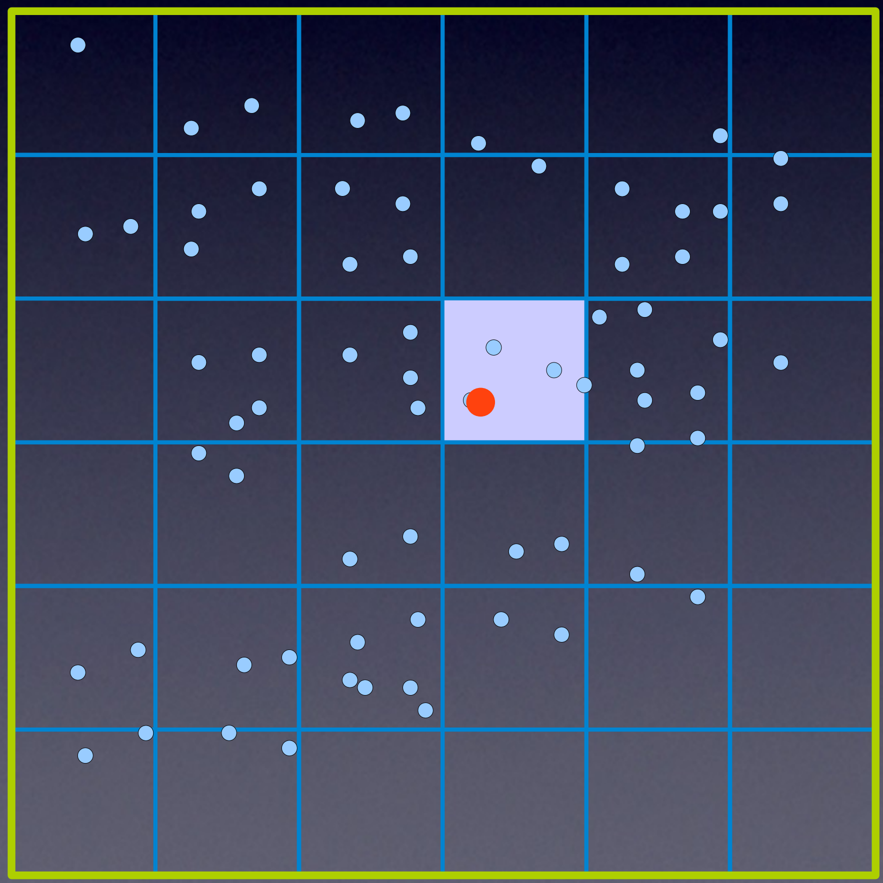
Uniform Grid

- Given a query point, how would this help?



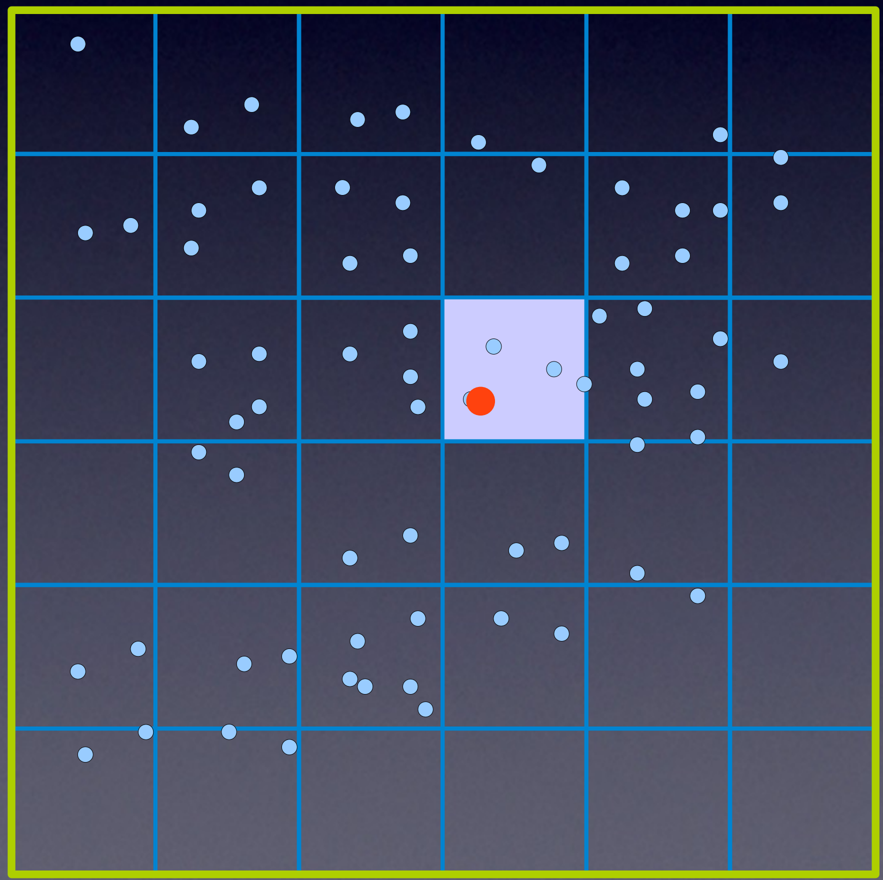
Uniform Grid

- Start from the cell that the query point belongs to



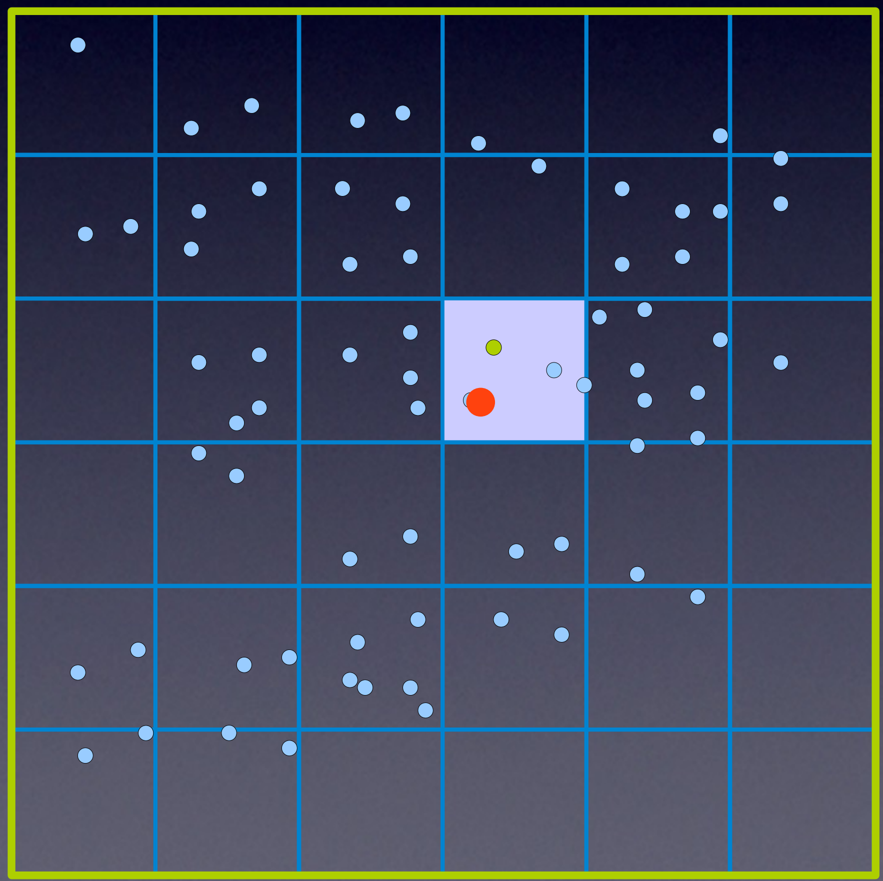
Uniform Grid

- Find the closest point



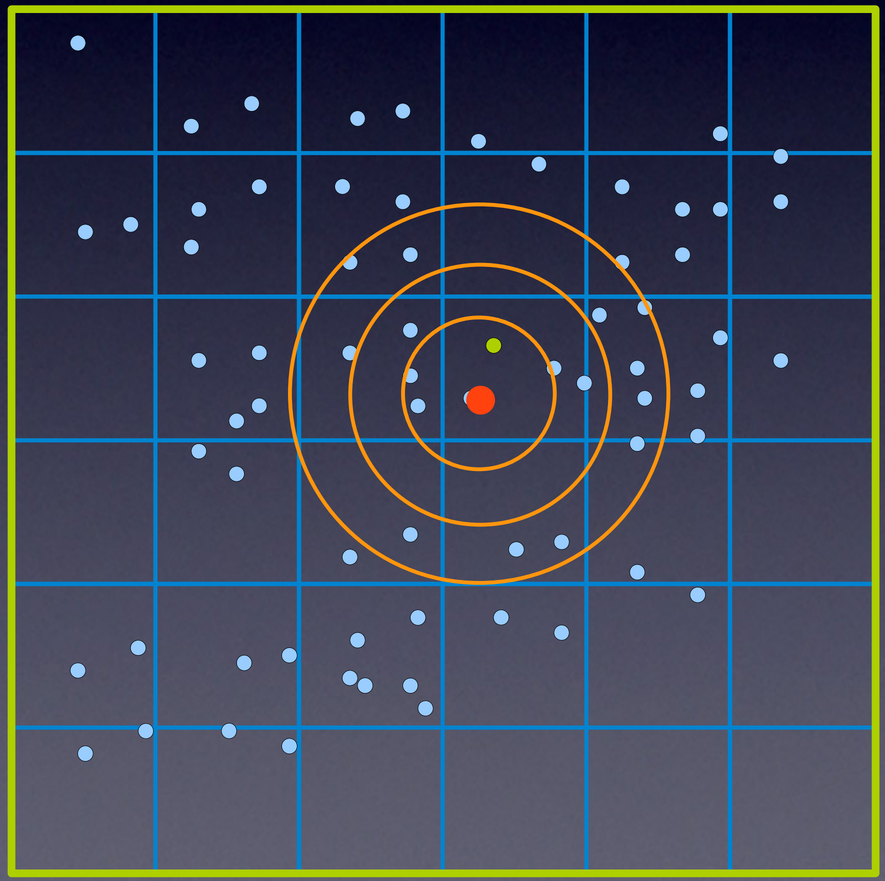
Uniform Grid

- Are we done yet? Why?



Uniform Grid

- Search is continued in concentric circles centered around the query point. All cells touching the circle must be searched.

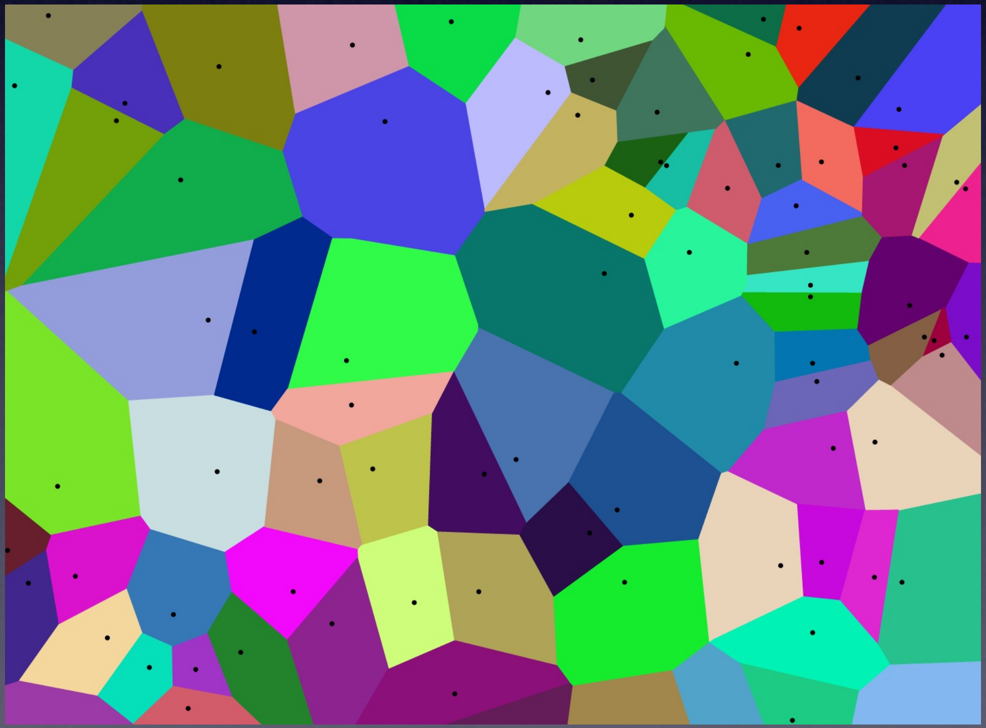


Uniform Grid

- GPU Implementation Steps

1) Build uniform grid

2) Search



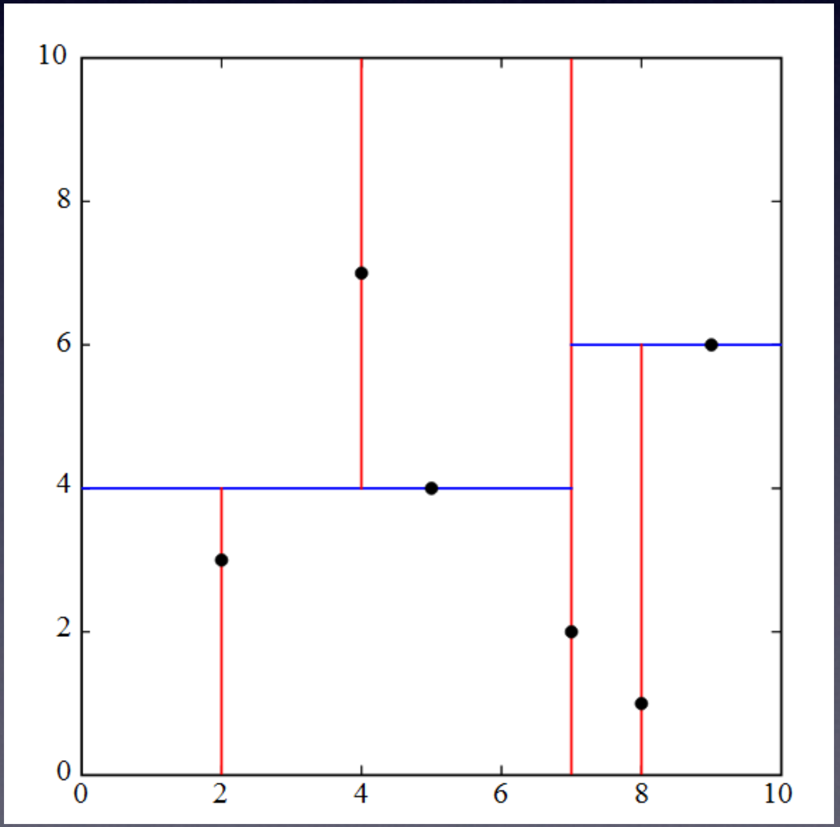
KD-Tree

- Short for *K-Dimensional Tree*.

  Use axis-aligned planes/lines to recursively split the original point set into subsets.

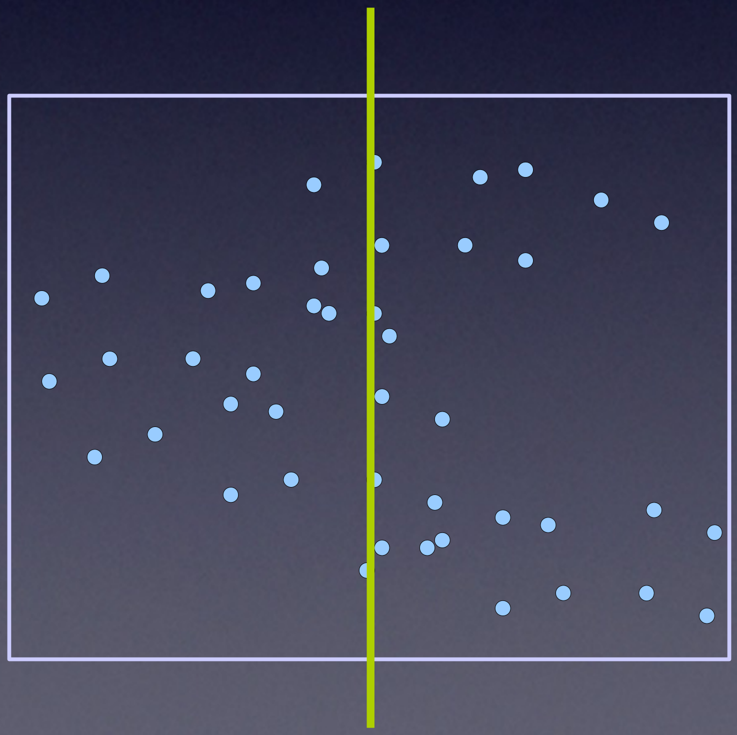
- Just like 'split', except this is in k-dimensional, so the each split is with respect to a possibly different dimension.

- Result in a binary tree.



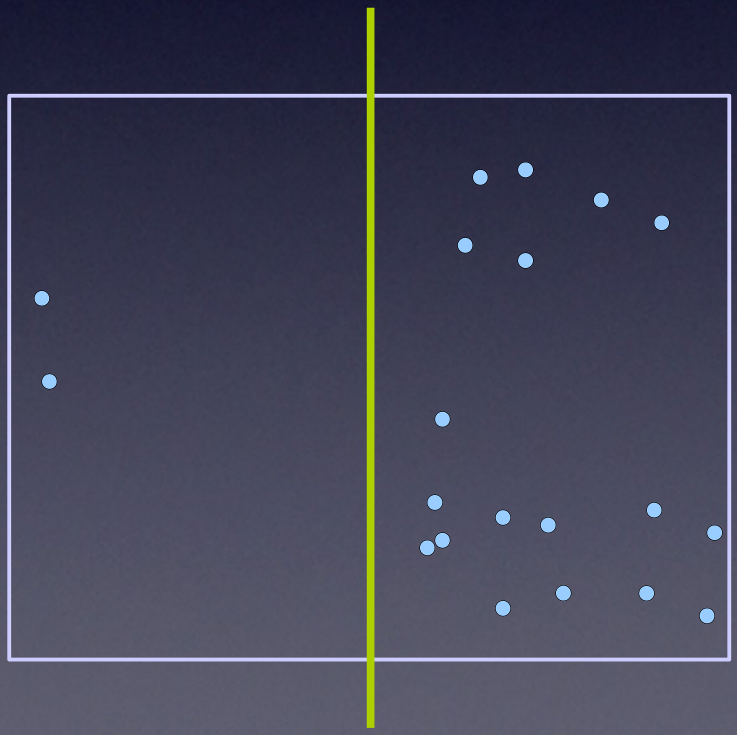
KD-Tree

- Where to put the splitting plane?
  - Mid-point Split: center of bounding box



KD-Tree

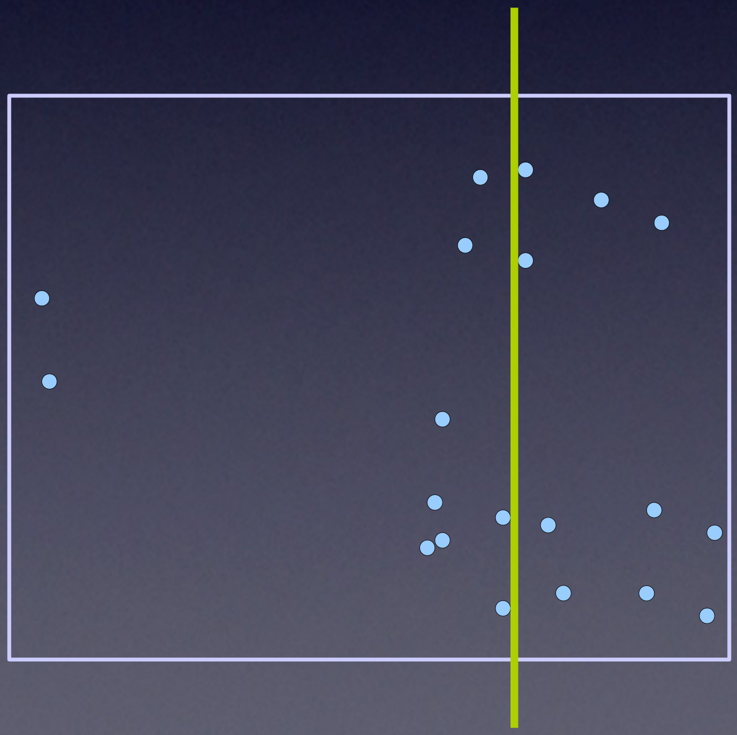
- Where to put the splitting plane?
  - Mid-point Split: center of bounding box
    - Does not work so well for non-uniform points



Unbalanced Tree

KD-Tree

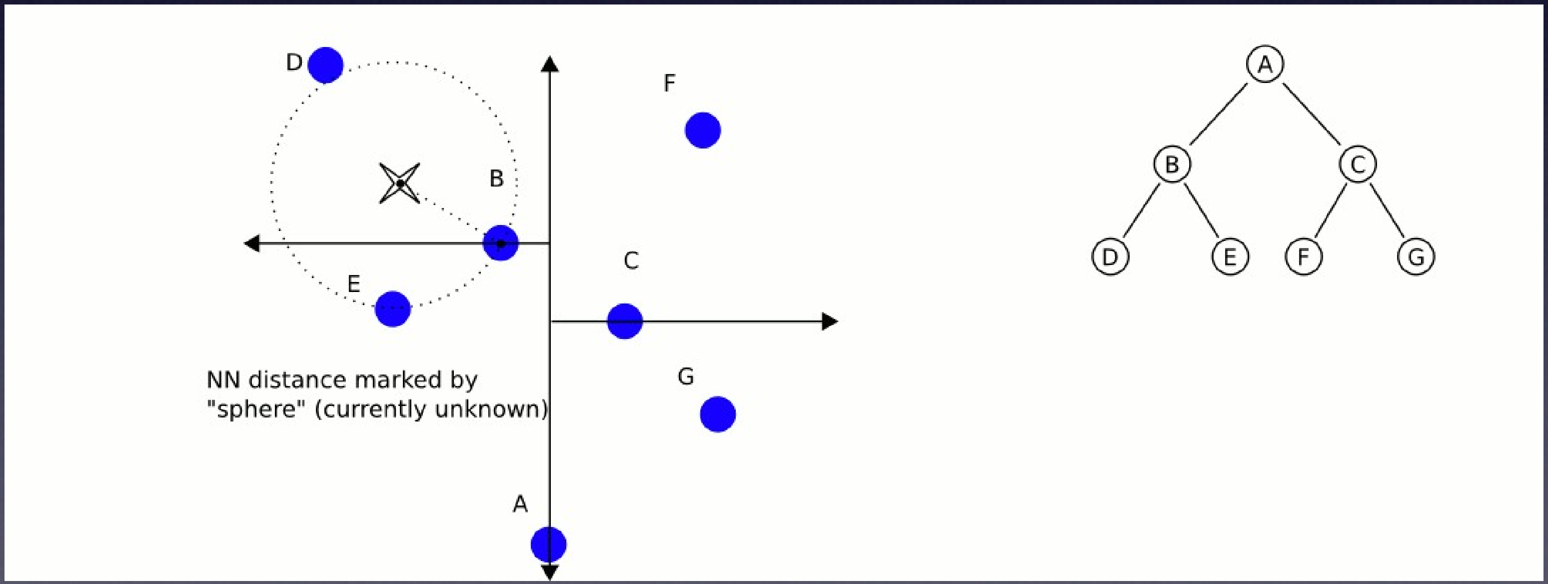
- Where to put the splitting plane?
  - Median Split: median of the current group of points
    - More expensive to compute



Balanced Tree

KD-Tree

- Search Algorithm
  - Given a kd-tree, how do we use it to search for the closest point to the query point?



KD-Tree

- GPU Implementation Notes

  1) Build kd-tree

  - Conceptually similar to quick sort, except much easier if only using middle-point split
  - Utilize segmented scan

  2) Search

  - Recursive, can use a manual stack maintained in shared memory or register space

kNN-Search

- Question: given a point set S, and an arbitrary point p in space, find for p the k closest points in S.
- NN search is equivalent to 1NN-search
- Sometimes also imposes a maximum search radius rmax.