Homework 2

1. A game for two players starts with an empty pile. Players take turns putting 1 or 2 or 3 pennies in the pile. The winner is the player who brings the value of the pile up to 16¢.
   (a) Make a directed graph modeling this game.
   (b) Show that the second player has a winning strategy by finding a set of four "good" pile values, including 16¢, such that the second player can always move to one of the "good" piles (when the second player moves to one of the good piles, the next move of the first player must be to a non-good pile, and from this position the second player has a move to a good pile, etc.).

2. Prove that no graph with two or more vertices has the property that all vertices have distinct degrees.

3. If the edges in the graphs below are considered to be undirected, are the two graphs isomorphic?

4. Which of the following pairs of graphs are isomorphic – obviously show your work.
   (a) 
   (b) 

5. If graph G has N vertices, all of which but one has odd degree, how many vertices of odd degree are there in G-bar, the complement of G?

6. Determine which of the following graphs is bipartite – show your work.

7. Show that these graphs: $K_{3,3}$, $K_5$, and $K_6$ can be drawn without crossing lines on the surface of a torus (a donut).

8. Determine if either of these sequences is graphic (and of course show your work)
   (6,6,5,5,2,2,2,2); (6,6,5,5,3,3,3,3).

9. If a graph has 50 edges, what is the minimum number of vertices it can have?